# An Inside Track: Fostering Mathematical Practices

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## *I admit it.*

*I was the teacher who initially thought games were filler activities that allowed me time to work with students who needed individual help. I viewed this as a win-win situation. My students reviewed concepts while I provided one-on-one instruction. But when I closely observed my students playing games, they weren't just reviewing. I realized they were using high-level thinking. Now I use games as an instructional tool to develop my students' mathematical understanding.* (Mr. Vaughn, a second-grade teacher)

Classroom teachers, like second-grade veteran teacher Vaughn, may not initially consider games as opportunities for students to engage in deep mathematical thinking. However, in this article, we reveal how Vaughn used Attribute Trains, a game he adapted from NCTM Illuminations (http://illuminations.nctm.org/) to foster his students' thinking related to key ideas within the Standards for Mathematical Practice (SMP) from the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010).

Conversations from a second-grade class demonstrate how teachers can use mathematical games to emphasize key ideas from the Common Core's Standards for Mathematical Practice.

## The game

Attribute Trains addresses students' understanding of patterns and attributes of geometric shapes. To play the game, students place two attribute blocks in the play area (e.g., a set of blocks including five different shapes, with each shape represented in two sizes and three different colors, for a total of thirty blocks in the shape set). One block, known as the *starting*, or *first*, block, is placed on the far left of the table. Another block, called the *ending*, or *last*, block, is placed on the far right. The players set a goal for the number of blocks needed to move from the first block to the last block, with the criteria that each attribute block can be different from the previous block in exactly one way only (e.g., shape, size, color) (see **fig. 1**). Players begin adding blocks to the train with the intention of meeting or exceeding the previously established goal. As each player adds a block to the train, he or she must explain why it can be added.

Any player can challenge another player's justification, and if the challenge is correct, the challenged player must remove the block and try again. Play continues until either no more blocks can be added to the train or the next block that can be added corresponds to the block in the final spot.

## Addressing content *and* practice standards

At first glance, the students in Vaughn's class apparently identified various shapes, used mathematical vocabulary, and attended to both defining and nondefining attributes as they engaged in the Attribute Trains game. The excerpt below highlights a conversation that occurred as students were introduced to the first rule of the game—changing one attribute with each move.

***Vaughn:*** The first block is a small, red square, and the next block on the train is a small, blue square. Who can help me think about why we call this shape [*holding up a small, red square*] a *square* instead of just a *rectangle*?

***Taylor:*** Squares have four sides that are the same.

***Grant:*** And they have square corners. See? [*Pointing to the right angles*] And if it was just rectangles, it would be not all sides equal.

***Vaughn:*** Remember, a square is a special rectangle with four equal sides and four right angles. Now we need to think about how we can move from the first blocks, which are squares, to the last block, which is a big, yellow hexagon. Which block do you think could come next on my train?

***Kieran:*** It's changing color. The next one will be a small, yellow square.

***Demetrius:*** No [*sorting through the pile of blocks*], I don't think it's gonna be a square. It's a yellow triangle next. We didn't do yellow yet.

***Taylor:*** You're right, D [*as Vaughn places a small, blue triangle in the train*]; it's a triangle, not a square! Three sides, not four.

FIGURE 1

The Attribute Trains game involves moving from first to last block by a single attribute.

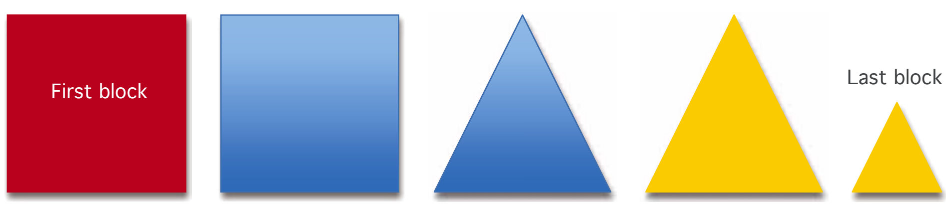
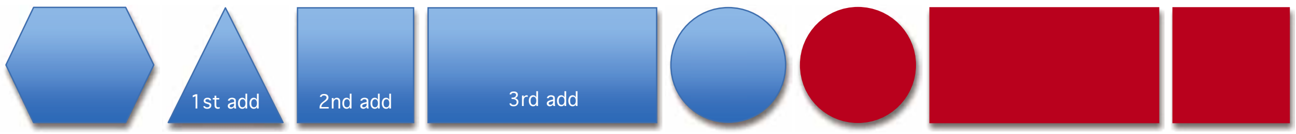


FIGURE 2

A student justified his accurate number of moves with this representation.




***Vaughn:*** You did a nice job predicting the next shape, even though the color is different [*pointing to the blue triangle that was added to the train*]. There are many ways to solve this problem. Kieran's small, yellow square could have worked, but why couldn't we use a yellow triangle?

In this excerpt, Kieran, Demetrius, Grant, and Taylor recognized shapes and specified attributes, such as the number of sides and the types of angles, which exemplifies Common Core State Standard 2G.1: *Reason with shapes and their attributes* (CCSSI 2010, p. 20). Throughout the brief dialogue, Vaughn posed guiding questions that prompted students not only to review the defining attributes of a square but also to consider the hierarchical relationship between the square and the rectangle. From the classroom observations and with further reflection on the vignette, it is apparent that the game also provided a means for students to engage in several of the Standards for Mathematical Practice (SMP) as they analyzed the shapes' attributes. Specifically, the students' conversations addressed SMP 1, 3, and 7 during the time that they played Attribute Trains.

## SMP 1

*Make sense of problems and persevere in solving them.*

In the previous excerpt, students used the attribute block trains as a concrete referent for their thinking, which helped them "conceptualize and solve a problem" (CCSSI 2010, p. 6). Moreover, the shape attributes served as constraints in the construction of the train. For example, when Demetrius predicted the yellow triangle as the next block in the train, Vaughn asked why that block could not follow the small, blue square. This guiding question provided an opportunity for students to "analyze givens, constraints, relationships, and goals" (p. 6) as they sorted through the set of blocks. The yellow triangle did not follow the constraints of the problem because two attributes (color and shape) would be different, rather than just one.

## SMP 3

*Construct viable arguments and critique the reasoning of others.*

In the first excerpt, Grant extended Taylor's definition of a square by adding that squares also have four right angles. As students continued to play the game, they "construct[ed] arguments [as they] justif[ied] their conclusions, communicate[d] them to others, and respond[ed] to the arguments of others" (CCSSI 2010, p. 6). In the next set of dialogue, Steven, Sara, Amit, and Emmalynne used differences in attributes to debate the definition of the number of moves.

***Sara:*** I predicted we could go for seven moves, and we went seven blocks.

***Steven:*** No, you start at the first block. You put seven more blocks [*see* **fig. 2**], but that's only six moves; see, because when you added your first block here [*pointing to the first additional block*] to here [*pointing to the second additional block*], it is one move. And then here [*pointing to the second additional block*] to here [*pointing to the third additional block*], that's another move. So that's only six moves.

***Emmalynne:*** But you forgot the first block. We had to pick our block because it had to be different in one way from that block. See? [*Pointing to the starting block*] It is a big, blue hexagon. Our block we picked first is a big, blue triangle. It's different because the shape is different.

***Amit:*** We can count our moves by how many blocks we add after the first one.

***Steven:*** That makes sense. We can count how many blocks we add, and that can be our moves. It will be the same either way. I just forgot to count the starting block.

In this conversation, Steven, Emmalynne, and Amit negotiated what they meant by *the number of moves* after communicating their thinking about where they should begin counting—from the first added block or the original block. Through the conversation with his peers, Steven recognized the need to start counting from the initial block (the blue hexagon) rather than the first block (the blue triangle) that was added. Moreover, they justified their conclusions by referring to concrete objects and using the attributes of the shapes in their arguments.

### SMP 7

*Look for and make use of structure.*

As the second graders in the previous excerpts played Attribute Trains, they "look[ed] closely to discern a pattern or structure" (CCSSI 2010, p. 8) within the arrangement of the blocks in each train to make sense of the context of the problem or to predict the next block in the sequence. The students also attended to the structures of the shapes and reasoned about attributes of the various shapes. The following dialogue represents how Vaughn's students attended to structure and used these observations to make conjectures regarding the number of moves necessary to complete the train.

***Vaughn:*** [*Putting up a small, red square on the left side of the whiteboard in the play area and a large, blue hexagon on the far right side of the play area*] How many blocks do you think we need to move from the small, red square [*pointing to it*] to the large, blue hexagon [*pointing to it*]?

***Laura:*** I think eight moves. No, five.

***Vaughn:*** Why did you change from eight moves to five?

***Laura:*** Well, I think we can do with eight, but I think we can do less, too.

***Vaughn:*** Draw the figures in your math journal. Add five dashes for the five blocks we are going to use to reach the large, blue hexagon [*demonstrating drawing the dashes between the first and last block*]. Label your shapes and talk to your partner about Laura's prediction. Why would five be a good number of moves to try?

[*Students begin to draw and talk to one another.*]

The time that students spent creating their pictorial representation of the first and last block gave them a moment to reflect on the structural differences between the two displayed shapes. For instance, after drawing the attribute blocks in their math journals, almost every one of Vaughn's twenty-three students verbally acknowledged that each of the attributes (e.g., shape, size, and color) were different in the two blocks. For this reason, the class came to the consensus that students would need several blocks to move from the first to the last block.

***Vaughn:*** [*After students have had time to draw and discuss*] Let's talk about our prediction. Why was five a good number of moves to estimate for this round? Do you think we can use less?

***Jess:*** Me and Mike said that five was pretty good because we have to change the shape and the color and make it big. That's a lot of things to do, so we need to have room to do that.

***Elliott:*** I think maybe three. 'Cause we only have to change three things.

***Vaughn:*** Hmmmm. We have some interesting ideas. One thing I noticed both Elliott and Jess said was that we had to change three attributes: shape, size, and color. Let's keep that in mind as we complete our train, and we can decide if we need more or less dashes.

[*Students work in pairs to complete the attribute train in five moves or fewer.*]

***Danny:*** I used five with our train, but we could have stopped earlier.

***Vaughn:*** What do you mean, Danny? Did anyone else feel like they could have stopped earlier?

[*Several students nod their heads.*]

***James:*** We thought so because first we changed the shape and then we changed the color [*pointing to a representation of his train*]. Then all we had to do was change the size to make it small.

***Elliott:*** I think I can do three blocks 'cause there are three different parts.

Here, Vaughn used guiding questions that encouraged students to (a) examine the structure of the attribute train, (b) confirm or refute their predictions, and (c) support their conclusions with evidence that occurred during the game. Prompting students with such questions as "Why was five a good number of moves to estimate for this round?" and offering opportunities for students to share conjectures about the relationship between the number of attribute differences and the number of moves allowed his students to examine the structure of the attribute train and analyze the situation before diving into the task. Moreover, the questions Vaughn used helped students focus on mathematical goals, such as generalizing the relationship between the difference in the number of attributes and the fewest number of moves necessary between the "engine" and the "caboose."

***Vaughn:*** Hmmmm. You made an interesting hypothesis, Elliott, about the relationship between the number of train cars and the number of attributes that are different. Let's make a table to organize our findings. Then we can use our table to see if our hypothesis always works [*drawing a table on the board*]. Let's see; we should keep track of the number of different attributes between the engines and the cabooses. And we need a column to record the least number of moves we find [*see* **table 1**]. Test it out. See what the least number of moves is that we need making a train from the small, red square to the large, blue hexagon. [*He gives students time to test*].

***Austin:*** Yes! It worked. And I found you can do three moves more than one way!

With guiding questions like these embedded throughout the activity, Vaughn gave his students opportunities to critically analyze the structure of the train (SMP 7), justify their thinking, and analyze the solutions of others (SMP 3)

as they persevered while attending to the constraints of a complex problem with multiple solutions (SMP 1).

**TABLE 1**

This table depicts the number of attribute differences and the fewest number of moves.

| Trial | Different attributes | Fewest number of moves |
|---|---|---|
| 1 | 3 | 3? |
| 2 | | |
| 3 | | |
| 4 | | |

## Extending students' mathematical thinking

The Attribute Trains game includes several variations. This flexibility presented an appropriate challenge based on students' individual needs and interests and allowed all students in Vaughn's class to engage in mathematical thinking related to the SMP. For instance, several students played the game without the constraint of the last block—working to extend the train as far as possible while changing only one attribute at a time. Others continued to play the game as they had in the examples, challenging their peers to find the fewest number of blocks needed to move from the first block to the last given block. Four students extended the activity further to test Austin's finding (from the previous dialogue) and see if there were multiple ways to arrange the trains with the fewest number of moves.

## A valuable use of class time

While Vaughn's second graders participated in the Attribute Trains game, they engaged in several of the SMP by—

1. persevering while solving problems with constraints;
2. justifying conjectures using physical, pictorial, and verbal representations of geometric shapes; and
3. examining structure.

As demonstrated in the conversations that emerged in Vaughn's classroom, incorporating mathematical games into the instructional period can be a valuable use of class time. In fact, play provides a context for students to explore and manipulate mathematical ideas when teachers incorporate higher-order discussion questions that engage children in reflection and representation (Dockett and Perry 2010; NAEYC/NCTM 2010; Olson 2007). Therefore, when teachers construct guiding questions that encourage students to "reflect on and represent the mathematical ideas that emerged in their play" (NAEYC/NCTM 2010, p. 8), they present students with opportunities to use critical thinking, while emphasizing content, all in the context of having fun.

**Common Core Connection**
**2.G.1**

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